Title: Actuator and sensor placement for neural fields

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Description of the Thesis

I. Context and scientific objectives

Neural fields are *nonlinear integro-differential equations* that model the activity of **neuronal populations** [1, 2]. They provide a **continuum approximation of brain structures**, inspired by the high density of neurons and synapses in the brain. The infinite-dimensional nature of neural fields allows for accounting for the spatial heterogeneity of neuronal activity and the complex synaptic connections between them. Additionally, their delayed versions can consider the non-instantaneous communication between neurons. Unlike numerical models that represent each individual neuron with a set of differential equations, neural fields are conducive to mathematical analysis. A wide range of mathematical tools is now available to **predict, analyze, and control their behavior**. These tools can assess the existence of stationary patterns [3, 4], perform stability analysis [5], and facilitate feedback stabilization [6].

This balance between biological significance and abstraction accounts for the **diverse applications of neural fields**, which span across various areas such as the primary visual cortex [7, 8], the auditory system [9], working memory [10], sensory cortex [11], and deep brain structures related to Parkinson's disease [6]. The advancement of modern technologies, such as *multi-electrode arrays* and *calcium imaging*, enables the measurement of neuronal activity with increasingly high spatial resolution. By utilizing these measurements to estimate the synaptic distribution among neurons, we can better understand the internal organization of specific brain structures. In our most recent research, we developed **adaptive reconstruction techniques** [12] to estimate the evolution of the system in real-time. This estimation of synaptic distribution could be instrumental in enhancing the feedback control of neuronal populations. A particularly relevant example of this is **deep brain stimulation** (DBS), which involves electrically stimulating deep brain structures associated with neurological disorders like Parkinson's disease [13]. Research has shown that stimulation proportional to the activity of a brain structure known as the subthalamic nucleus is sufficient to disrupt brain oscillations associated with Parkinson's disease [6].

Although these estimation and control strategies represent significant theoretical breakthroughs, they often rely on **stringent assumptions**, such as requiring that the neuronal population be measured or actuated throughout the entire spatial domain. In practical applications, however, only a **limited number of local actuators and sensors** are available. This limitation can raise important questions regarding observability and controllability. For example, depending on the kernel that governs neuronal interconnections, one might wonder what information is necessary to reconstruct the entire spatiotemporal state of a brain structure, or at the very least, to obtain a significant spatial average of that state. Furthermore, in systems composed of **different neuronal populations**, only certain populations can be effectively actuated or measured. In the context of Parkinson's disease, one may wonder how this limited information can be used to successfully compensate for pathological neural oscillations.

Thesis goal: **The general objective** of this thesis is to analyze the impact of actuator and sensor placement on the control and estimation of neural fields comprising multiple neural populations. Specifically, we aim to address the following three questions:

1. How can electrodes be placed to ensure the observability and controllability of the neural field?
2. How can electrode placement be optimized to enhance brain stimulation effects or to gather the most informative data on brain structures?
3. How can we design effective output-feedback control laws to achieve selective disruption of pathological neural oscillations?

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II. Scientific approach

To address the challenging questions of **controllability and observability** for neural fields, we propose two approaches. The first approach involves **projecting** the kernel appearing in the integro-differential equations onto a functional basis to obtain a finite-dimensional system . Then, we will adjust appropriate techniques developed for finite-dimensional systems to assess controllabilty and observability of this reduced model. In particular, it will be possible to use **optimal control methods** to effectively position the available actuators and sensors in the spatial domain. The main challenge here is to ensure that the terms neglected during this projection do not undermine the observability/controllability of the original infinite-dimensional system, which is known as the spill-over effect [14]. The second approach is more **structural**; it involves leveraging the specific structure of the kernel to establish intrinsic conditions for observability, similar to the Hautus criterion (see, for example, [15]). These intrinsic conditions will facilitate the use of invertible integral transformations, thereby simplifying the design of stabilizing control laws and observers for the infinite-dimensional system. The principal steps of the proposed work are listed as follows:

- We will first consider a **single neural population** and simplify the dynamics through appropriate **linearization**. This will allow for the development of suitable actuator and sensor placement methods for this simplified infinite-dimensional system.
- We will extend the proposed approach to **non-linear neural fields** or neural fields with **multiple neural populations**. In this case, we may encounter underactuated systems due to the inability to act on all populations.
- As an application test case, we will focus on designing an output-feedback law to selectively **disrupt pathological neural oscillations**. We envision utilizing a network of interconnected systems, with each system generating oscillations in a specific frequency band. Since each subsystem will receive the same control inputs, disrupting pathological oscillations while leaving healthy ones unaltered presents a significant challenge. Our goal is to inhibit the activity of neuronal circuitry involved in the generation of pathological oscillations while preserving the activity of the rest of the structure. This objective will request the robustness assessment of limit cycles, which is often a complicated task [16].
- Finally, we aim to showcase the efficiency of our proposed approaches through **numerical simulations** on a detailed numerical model of the brain structures. We will deploy, demonstrate, and validate our theoretical findings across various neural fields configurations.

III. Research environment and potential collaborations

Pôle Systèmes of L2S is one of the leading groups in control theory worldwide. It has been developing control theory methodologies for neuroscience applications for approximately ten years. In collaboration with neurosurgeons from H. Mondor Hospital (S. Palfi and S. Senova), neuroscientists from NeuroPSI (A. Destexhe), and control researchers (M. Sigalotti at Inria, M. Lowery at University College Dublin), we have created innovative closed-loop strategies for Deep Brain Stimulation. These strategies aim to disrupt pathological brain oscillations associated with Parkinsonian symptoms. As a result, collaborations with these teams will be envisionned during the thesis.

IV. Required skills

This thesis topic requires strong skills in control systems and mathematics, ideally at the level of Grandes Écoles or a Master's degree in mathematics or control. Excellent performance in the engineering curriculum, as well as expertise or a keen interest in topics related to automation and neuroscience, will be advantageous for this proposed subject. The research will facilitate the acquisition of advanced skills in neural fields, integro-differential equations, and numerical implementation. The candidate will also need to become proficient in Julia, Matlab, or Python, particularly in numerical methods and simulations.

V. Application

To apply, write an email with your CV and a transcript to Jean Auriol, Lucas Brivadis and Antoine Chaillet: `firstname.lastname@centralesupelec.fr`. The thesis is conditional upon acceptance by the STIC Doctoral School and is expected to begin in October 2026.

VI. References

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Advisors' Biographies

Jean AURIOL (thesis supervision load: 33%). He received his Master's degree in civil engineering in 2015 (major: applied maths) from MINES ParisTech, part of PSL Research University, and in 2018 his PhD degree in control theory and applied mathematics from the same university (Centre Automatique et Systèmes). From 2018 to 2019, he was postdoctoral researcher at the Department of Petroleum Engineering, University of Calgary, Canada, where he was working on implementing backstepping control laws to attenuate mechanical vibrations in drilling systems. Since December 2019, he has been an associate researcher (Chargé de Recherche) at CNRS, Université Paris-Saclay, CentraleSupélec, L2S. His research interests include robust control of distributed parameter systems and interconnected systems.

Lucas BRIVADIS (thesis supervision load: 33%). He received in 2018 his engineering degree from École Centrale de Lyon and his master's degree in applied mathematics from Université Lyon 1. He defended his PhD in 2021 at LAGEPP, Université Lyon 1. From 2021 to 2022, he was a postodcotral researcher at L2S, (CNRS, CentraleSupélec, Université Paris-Saclay). Since 2022, he is a CNRS researcher (chargé de recherche) at L2S. His research interests focus on infinite-dimensional observer design for hyperbolic PDEs under weak observability assumptions (with applications to crystallization processes) and output feedback stabilization of nonlinear systems with observability singularities.

Antoine CHAILLET (thesis supervision load: 34%). Antoine Chaillet received his B.Sc. degree from ESIEE Amiens in 2002, and his M.Sc. degree in Control Engineering from Univ. Paris Sud 11 in 2003. In 2006, he received his Ph.D. degree cum laude in Control Theory from Univ. Paris Sud 11-L2S. In 2004, he was recipient of a Marie-Curie Scholarship to visit Università degli Studi di Firenze, Italy. In 2006–2007, he served as a post-doc fellow at Centro di Ricerca Piaggio, Pisa, Italy. From 2007 to 2016, he served as an associate professor at L2S-Univ. Paris Sud 11-Supélec-EECI. He is now full professor at CentraleSupélec and former junior member of Institut Universitaire de France. His research interests include stability analysis and stabilization of nonlinear systems, time-delay systems, and control theory for neuroscience. He is the author of around 100 peer-reviewed publications on these topics.